17-1 Random Variables

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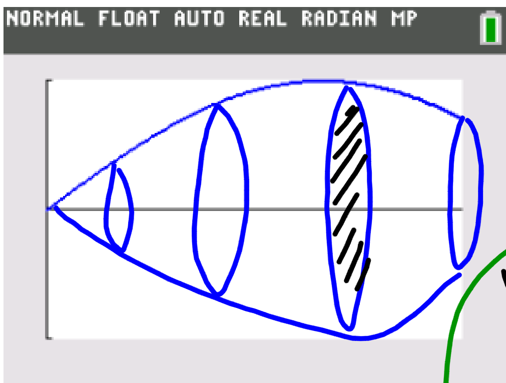
15. The graph of $y = \sin(3x)$ for $0 \le x \le \frac{\pi}{4}$ is rotated through 2π radians about the x-axis.

Find the **exact** volume of the solid of revolution formed.



$A = \pi r^2$

$A = \pi(\sin(3x))^2$

$V = \pi \int_0^{\pi/4} \sin^2(3x)\,dx$

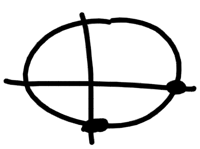
$\cos(2\theta) = 1 - 2\sin^2\theta$

$\frac{\cos(2\theta) - 1}{-2} = \frac{-2\sin^2\theta}{-2}$

$\frac{1-\cos(2\theta)}{2} = \sin^2(\theta)$

$= \pi \int_0^{\pi/4} \frac{1-\cos(6x)}{2}\,dx$

$= \frac{\pi}{2} \int_0^{\pi/4} 1 - \cos(6x)\,dx$



$= \frac{\pi}{2}\left[x - \frac{\sin(6x)}{6}\right]_0^{\pi/4}$

$= \frac{\pi}{2}\left[\left(\frac{\pi}{4} - \frac{\sin\frac{3\pi}{2}}{6}\right) - \left(0 - \frac{\sin 0}{6}\right)\right]$

$= \frac{\pi}{2}\left[\left(\frac{\pi}{4} + \frac{1}{6}\right) - (0)\right]$

$= \frac{\pi}{2}\left(\frac{\pi}{4} + \frac{1}{6}\right) = \frac{\pi^2}{8} + \frac{\pi}{12}$

A random variable is classified as discrete if its set of possible values are isolated points on the number line – there are a countable number of possible values for the variable.

A random variable is classified as continuous if its set of possible values is an entire interval on the number line – it can take on any value on the interval. There is an uncountable number of possible values for the variable.

Ex1. State whether each of the following is a discrete or continuous random variable.

1.) The number of new bicycles sold each year by a bicycle store.

2.) The volume of water in a reservoir.

3.) The number of defective light bulbs in a batch.

4.) The weight of a student.

5.) The length of hair on a horse.

6.) The number of leaves on the ground in the fall in Minnesota.

Discrete Probability Distributions

Ex1. 4 coins are tossed and the number of heads is recoded.

a.) Find the probability of each possible outcome.

b.) Create a frequency bar graph that shows the probabilities of the possible outcomes.

X = the # of heads

x	P(x)
0	1/16
1	4/16 = 1/4
2	6/16 = 3/8
3	4/16 = 1/4
4	1/16

# of outcomes $2^4 = 16$

Probability of each outcome $(\frac{1}{2})^4 = \frac{1}{16}$

HHHH

HHHT HHTH HTHH THHH

HTTH HHTT HTHT THHT THTH TTHH

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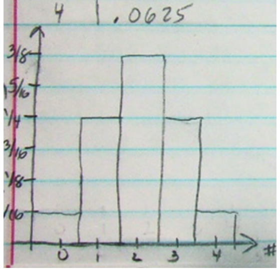
x	P(x)
0	.0625
1	.25
2	.375
3	.25
4	.0625



This is called either:

Probability Distribution Function (PDF)

Probability Mass Function (PMF)

A graph, function, or table that details all of the probabilities for all of the possible outcomes.

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The probability distribution for a discrete random variable is a table, graph, or formula that gives the possible values of X, and the probability P(X=x) associated with each value of x and follows the two rules:

$$0 \leq P(x) \leq 1$$

$$\sum_{x} P(x) = 1$$

The cumulative probability distribution of a random variable x expresses the probability that X does not exceed the value of x. That is:

$$P(X \leq x) = \sum_{y \leq x} P(y)$$

This is also called:

Cumulative Distribution Function (CDF)

Cumulative Probability Function (PMF)

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Ex1. 4 coins are tossed and the number of heads is recoded.

c.) Create a cumulative frequency table.

x	$P(X \le x)$
0	$1/16 = .0625$
1	$5/16 = .3125$
2	$11/16 = .6875$
3	$15/16 = .9375$
4	$16/16 = 1$

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Expected Value

The population mean, which measures the average value of X in the population, is also called the expected value of the random variable x. It is the value that you would expect to observe on average if you repeated the experiment an infinite number of times.

Ex1. 4 coins are tossed and the number of heads is recoded.

d.) Find the expected value.

$EV = \frac{1}{16} \cdot 0 + \frac{1}{4} \cdot 1 + \frac{3}{8} \cdot 2 + \frac{1}{4} \cdot 3 + \frac{1}{16} \cdot 4$

$P(x=0)$ $x=0$ $P(x=1)$ $x=1$

$EV =$

$EV = 2$

Let X be a discrete random variable with probability distribution P(x). The mean or expected value of x is given by:

$$\mu = E(x) = \sum x \cdot P(x)$$

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Ex2. A local church conducts a survey to determine the number of children each family has. Below are the results.

Number of Children	0	1	2	3	4	5	6	7
Frequency	14	18	13	5	3	2	2	1

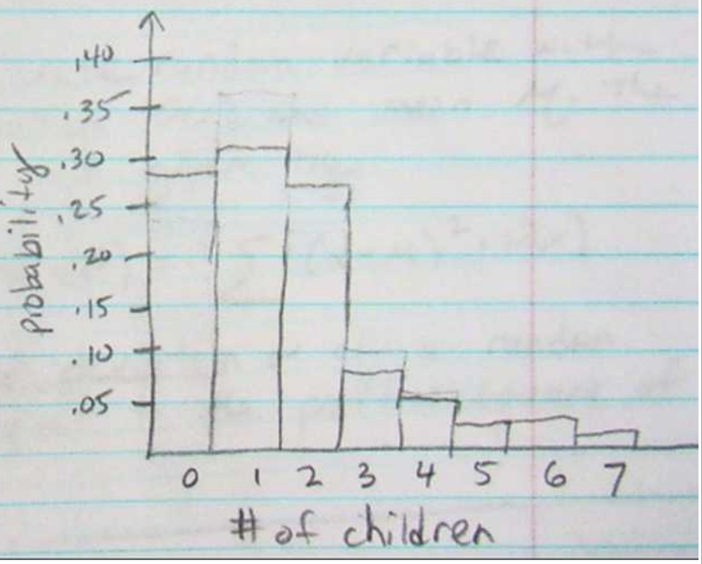
a.) Create a frequency distribution table and graph for the data.

b.) Create a cumulative distribution table.

c.) Calculate the expected Value

X = # of children

X	P(x)
0	14/58 ≈ .2414
1	18/58 ≈ .3103
2	13/58 ≈ .2241
3	5/58 ≈ .1042
4	3/58 ≈ .0517
5	2/58 ≈ .0345
6	2/58 ≈ .0345
7	1/58 ≈ .0172



$E(x) = 0 \cdot 14/58 + 1 \cdot 18/58 + 2 \cdot 13/58 + 3 \cdot 5/58 + 4 \cdot 3/58 + 5 \cdot 2/58 + 6 \cdot 2/58 + 7 \cdot 1/58$

≈ 1.724 children

Variance and Standard Deviation

Let X be a discrete random variable with probability distribution P(x) and mean μ. The variance of x is given by

$$\operatorname{var} x = \sigma^2 = E\left((x-\mu)^2\right) = \sum (x-\mu)^2 \cdot P(x)$$

and the standard deviation σ of a random variable x is equal to the positive square of the variance.

Ex2. A local church conducts a survey to determine the number of children each family has. Below are the results.

Number of Children	0	1	2	3	4	5	6	7
Frequency	14	18	13	5	3	2	2	1

d.) Find the variance and standard deviation.

x	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2 P(x)$
0	1.724	2.972176	.7174217931
1	.724	.524176	.162675
2	.276	.076176	.017074
3	1.276	1.62818	.14036
4	2.276	5.18018	.26794
5	3.276	10.7322	.370075
6	4.276	18.2842	.630489
7	5.276	27.8362	.479934

+

= 2.78597 = var x

σ = 1.66912

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